Worst Case Analysis Methods for Electronic Circuits and Systems to Reduce Technical Risk and Improve System Reliability

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Worst case analysis is one of the important elements needed to ensure reliable operation of electrical and electronic circuits and systems. Electrical system reliability and safety are critical for all applications where high reliability, long life, or operation under severe environmental conditions are important, including Defense, space, and automotive systems. This paper presents proven methods for performing worst case analysis of electrical circuits and systems and innovative worst case modeling methods.

Design margin and worst case analysis

Definition of design margin

Worst case analysis techniques are used to determine the design margin for electrical circuits and systems. The design margin, analogous to the margin of safety determined analytically for mechanical structures, is a measure of the margin between the worst case performance of an electrical system's design implementation and the performance required by specification. A positive design margin indicates that the electrical system will always perform as required with margin to spare. A negative design margin indicates that there are some conditions under which the electrical system will not meet its specification.

Determining design margin by worst case analysis

Analytically determining the design margin of electrical circuits and systems is the only way to verify that they will function properly under all conditions. Worst case performance is determined by analytically selecting the worst combination of conditions that the design can experience during its operational lifetime. In this way the worst case analysis produces a powerful analytical prediction of the variation of electrical design performance based on the known characteristics of the design and the demanding environments in which it must operate.

Importance of worst case analysis for high-reliability electronic systems

Many procurement activities for high-reliability electronic systems only require the seller to verify compliance with specifications by test, giving neither the designing nor the procuring agency insight into the design margin of the delivered hardware. Reliability analyses generally assume that electrical systems work properly and meet their specifications under all conditions, so not performing worst case analysis places electronic systems at risk for an unidentified quantity of reduced system reliability.

Worst case analysis methods for electrical circuits and systems

Analysis tools

Hand analysis

Hand calculation methods, using pencil, paper, calculators, and possibly computer spreadsheets, can be used for worst case analysis of electrical circuits and systems. The complexity of circuits to be analyzed using this method is generally limited, but gaining a qualitative understanding of circuit operation to verify the rationality of any chosen analytical method is *always* recommended.

Computer simulation programs

The most powerful methods for performing worst case analysis of electrical circuits and systems use computer simulation programs such as Spice (Simulation program with integrated circuit emphasis). Spice is a well-established industry-standard electronic circuit simulation tool that can produce accurate high-fidelity simulations of electronic circuit operation. An example of a Spice simulation output plot is shown in Figure 1. Spice and other computer simulation programs in conjunction with sound analytical methods are recommended for performing worst case analysis of electrical circuits and systems. They are powerful tools that can generally analyze more complex circuits with greater accuracy compared to hand analysis methods.

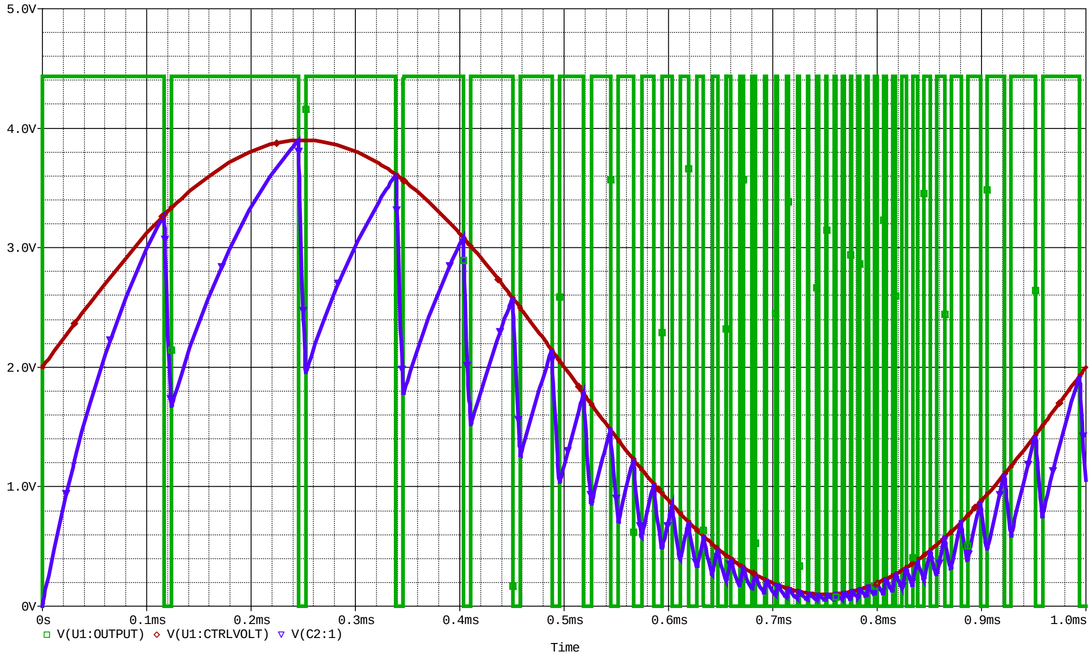


Figure 1 – Example of a Spice simulation plot

Factors affecting worst case performance

Worst case analysis must consider the following factors and conditions to determine circuit worst case performance, regardless of the analysis tools and methods used.

Factors affecting worst case performance	Examples
Part tolerance and parameter variation	Initial tolerance %, Minimum/maximum values (generally specified at room temperature)
Environmental conditions	Parameter variation with extremes of temperature, thermal shock, moisture, etc.
Aging effects	Parameter variation over operational life
Input and output interface conditions	Input voltage variations, Output load variations

Table 1 – Factors affecting worst case performance

Some of these factors are difficult or impossible to take into account during test, especially part tolerance and parameter variation, so *worst case performance cannot be determined by test*.

Recommended methodologies for worst case analysis

A worst case analysis uses the factors in Table 1 to determine the worst case parameters for each of the parts and interface conditions for an electrical circuit. Different types of worst case analysis can be performed depending on how these worst case parameters are adjusted in the circuit.

<u>Extreme value analysis</u>
Adjustment of all the parameters to their worst case values to produce the worst case circuit output characteristic results in the most conservative worst case analysis, called an extreme value analysis. A properly performed extreme value analysis will show what the performance would be at the outer edge of probability. If a positive design margin results from performing an extreme value analysis, the design is good.

If a positive design margin does *not* result from performing an extreme value analysis, it is usually appropriate to perform a Monte Carlo analysis to determine the probability of a negative design margin.

<u>Monte Carlo analysis</u>
Random adjustment of the parameters by a computer simulation program within the worst case limits produces a statistical analysis, called a Monte Carlo analysis. A Monte Carlo analysis, when used properly, can give the probability of a circuit output characteristic being within a given range, including determining the probability of a negative design margin. Monte Carlo analysis results are commonly shown in the form of a histogram. An example is shown in Figure 2. A histogram shows the expected probability distribution of a selected output characteristic under all defined operating conditions.

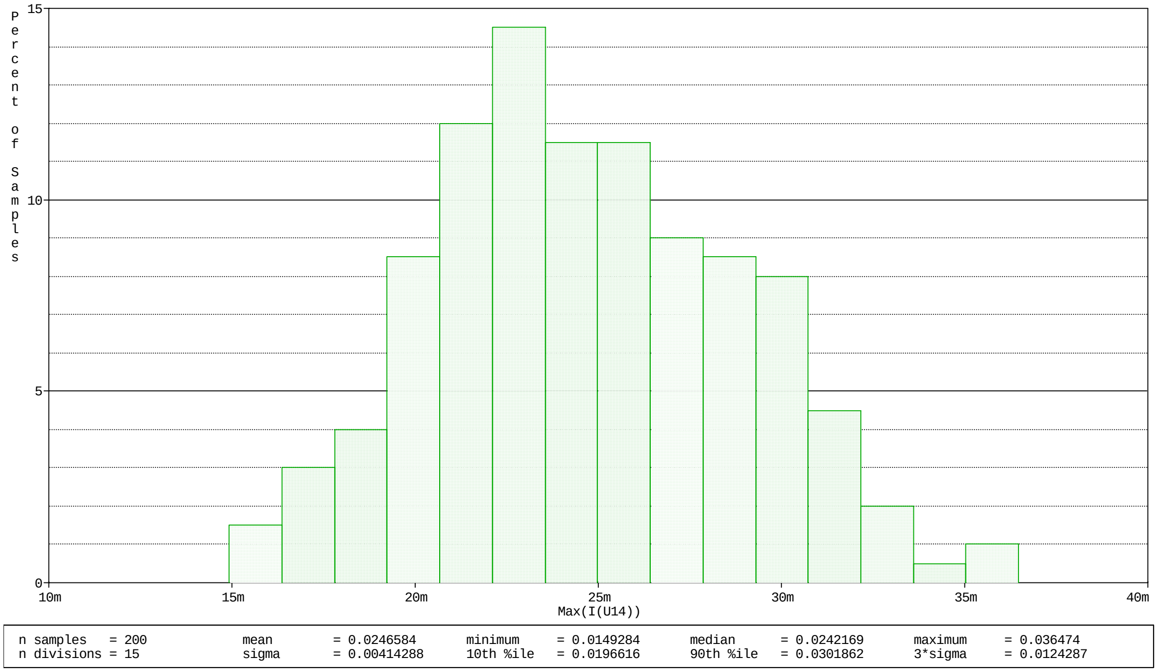


Figure 2 – Example of Monte Carlo histogram

Part models for worst case analysis

All of the factors affecting worst case part performance must be taken into account to produce worst case part models. Part model accuracy is the most important factor for achieving accurate analysis and simulation results.

Worst case (extreme value) models

The following guidelines are recommended for generating worst case (extreme value) models for all types of parts, including linear parts (resistors, capacitors, and inductors), diodes, transistors, MOSFETs, and macromodels (including op amps). These worst case modeling principles apply regardless of the choice of analysis tools, but will mainly be discussed for application to computer simulation.

Linear parts

Linear parts, including resistors, capacitors, and inductors, have a single primary characteristic to model. Different factors affecting the worst case value are usually considered to be independent of one another (temperature is independent of tolerance, etc.). Statistical mathematics shows that the most realistic way to combine the effects of independent factors is by using the root-sum-square (RSS) method instead of just summing them. The RSS method can be applied by the following formula, where T represents the calculated worst case tolerance due to all factors and $t1$, $t2$, $t3$ etc. represent the independent tolerance factors due to initial tolerance, temperature, aging, etc.

$$T = \sqrt{t1^2 + t2^2 + t3^2 + \ldots}$$

For example, specification MIL-R-55342F requires testing of chip resistors to verify maximum resistance variation due to several different independent factors. Table 2 shows an example of how these factors are combined using the RSS method to produce a realistic worst case tolerance.

Parameter	Symbol	Maximum value
Initial tolerance	*Tol*	1 %
Temperature coefficient	*TC*	100 ppm/°C
Life	*TL*	0.5 %
Moisture resistance	*TM*	0.5 %
Bonding exposure	*TB*	0.25 %
High temperature exposure	*TH*	0.5 %
Short time overload	*TO*	0.25 %
Low temperature operation	*TLT*	0.25 %
Thermal shock	*TT*	0.5 %
Calculated worst case RSS tolerance: $T = \sqrt{Tol^2 + (TC \cdot \Delta T)^2 + TL^2 + TM^2 + TB^2 + TH^2 + TO^2 + TLT^2 + TT^2} = 1.6\%$		

Table 2 – RSS tolerance calculation for MIL-R-55342F resistors

Some linear parts, particularly capacitors, may have non-uniform tolerances (e.g. +10%, -20%). This creates a bias (or offset) between the center of the worst case tolerance and the nominal value.

<u>Diodes</u>

Simple worst case tolerances that are effective for linear parts usually don't work for diodes and other semiconductor parts. Some exceptions could apply for hand analyses, but for Spice models the complex interaction of model parameters generally makes the use of simple model parameter tolerances impractical for worst case analysis. Specialized minimum and maximum worst case Spice models are recommended instead, using the guidelines in the following sections. The resulting worst case models can be used to produce accurate worst case analyses for most scenarios. Many Spice programs include part modeling utilities that can be applied using these guidelines to generate worst case models. Some additional specialized worst case models, such as maximum leakage current models, may be required for special cases.

Specialized minimum worst case diode models should have minimum output and speed. Specialized maximum worst case diode models should have maximum output and speed.

Diode characteristics	Worst case minimum model	Worst case maximum model
Forward voltage (Vf)	Maximum	Minimum
Junction capacitance	Maximum	Minimum
Switching times	Maximum	Minimum

Table 3 – Worst case diode part model guidelines

Transistors (bipolar)
Specialized minimum worst case transistor models should have minimum output, require maximum input, and have minimum speed. Specialized maximum worst case transistor models should have maximum output, require minimum input, and have maximum speed.

Transistor characteristics	Worst case minimum model	Worst case maximum model
Current gain (hFE)	Minimum	Maximum
Base-emitter voltage (Vbe)	Maximum	Minimum
Collector-emitter saturation voltage (Vce(sat))	Maximum	Minimum
Junction capacitances	Maximum	Minimum
Switching times	Maximum	Minimum
Bandwidth	Minimum	Maximum

Table 4 – Worst case transistor part model guidelines

MOSFETs
Specialized minimum worst case MOSFET models should have minimum output, require maximum input, and have minimum speed. Specialized maximum worst case MOSFET models should have maximum output, require minimum input, and have maximum speed.

MOSFET characteristics	Worst case minimum model	Worst case maximum model
Transconductance (gFS)	Minimum	Maximum
Threshold voltage (Vth)	Maximum	Minimum
On resistance (Rds(on))	Maximum	Minimum
Turn-on charge and capacitances	Maximum	Minimum
Switching times	Maximum	Minimum

Table 5 – Worst case MOSFET part model guidelines

Macromodels
Many of the more complex Spice models are created as macromodels, models made by putting together a number of more basic models and mathematical functions. Macromodels can be quite complex and varied, but the general guidelines for worst case macromodels are similar to other Spice worst case models. Minimum worst case macromodels should have minimum output, require maximum input, and have minimum speed. Maximum worst case macromodels should have maximum output, require minimum input, and have maximum speed.

Macromodel characteristics	Worst case minimum model examples	Worst case maximum model examples
Output	Minimum output voltage swing and gain; maximum output resistance	Maximum output voltage swing and gain; minimum output resistance
Input	Maximum input voltage threshold or offset	Minimum input voltage threshold or offset
Speed	Maximum switching times; minimum slew rate and bandwidth	Minimum switching times; maximum slew rate and bandwidth

Table 6 – Worst case macromodel general guidelines

The most widely used macromodels are for operational amplifiers (op amps) and comparators. Critical op amp characteristics include input offset voltage (Vos), open-loop voltage gain (Av), frequency response, and slew rate. Effects of op amp and comparator characteristics on circuit performance must be considered on a case-by-case basis and can be negligible depending on the circuit design. Worst case offset voltage often has the greatest effect on circuit performance and can be modeled as a simple voltage variation.

Monte Carlo models

To perform a Monte Carlo analysis, each variable circuit characteristic must have a probability distribution defined for it.

<u>Linear parts</u>
For linear parts a probability distribution must be defined for the primary characteristic using a probability distribution function with the worst case tolerance defining the edges of the distribution. Some commonly used probability distribution functions are shown in Table 7.

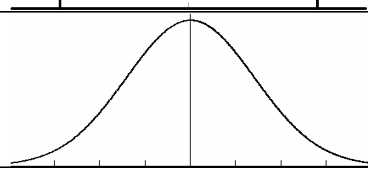
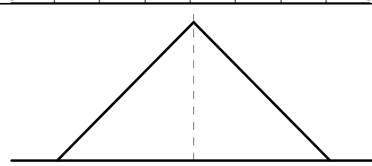
Probability distribution function	Shape	Application
Uniform		Simplest and most conservative – all values equally likely
Gaussian (normal)		Most realistic for most parameter variations
Triangular		Simple linear approximation of Gaussian, works well for non-uniform variations

Table 7 – Common probability distribution functions for Monte Carlo analysis

Complex non-linear parts

Complex native Spice models, including diodes, transistors, and MOSFETs, usually cannot use direct modeling methods for Monte Carlo analysis because of the complex interaction of their model parameters. Macromodeling methods can sometimes be used to linearly approximate the range of worst case operation for a Monte Carlo analysis.

Conclusion

Worst case analysis can be used to reduce technical risk and improve system reliability of critical electronic systems.

This paper has demonstrated:

- Proven methods for performing worst case analysis of electrical circuits and systems
- Innovative worst case modeling methods